

# Magnetic Coupling in Transmission Lines and Transformers

Explore the similarities of these closely-related structures and discard some widespread misunderstandings.

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During my investigation of transformers and transmission lines and the marriage of the two, a number of questions arose that I could not answer. For instance:

- If the characteristic impedance of a transmission line is the familiar  $Z_0 = \sqrt{L/C}$  (square root of inductance per length over capacitance per length), why doesn't this change when the line is surrounded by ferrite, which increases inductance?
- One source stated that the electrical length of a line is increased by adding ferrite, thus extending the low-frequency response. But electrical length also depends on per-length inductance (electrical length is physical length times frequency times  $\sqrt{LC}$ ), and my measurements showed that neither impedance nor electrical length vary with ferrite loading.
- What exactly are "conventional transformer currents" and what is the difference between this and "transmission line mode"?
- How can "choking reactance" of a transmission line transformer affect differential transmission line signals? This should be strictly a common-mode effect.
- Typically, transmission line transformers are wound with something like a half meter of line. At HF and even VHF this is a small fraction of a wavelength, so how can they be considered transmission lines at all? How can they develop significant longitudinal voltage if they are electrically short lines?

- If two wires carrying differential current pass through a core, the flux they produce cancels, resulting in zero flux in the core. Yet the core has a significant effect on low-frequency behavior. How can this happen with no flux?

I slowly pieced together explanations for those and other aspects of lines and transformers, which I haven't seen in the literature; perhaps you will find them helpful too. Through most of this discussion I will be referring to transmission lines as two wires, but the same ideas apply to any form of line except coaxial cable, which is treated separately.

## The Basics

First, I present a review of terminology and the basics. An isolated wire carrying a current,  $I$ , produces a magnetic field  $H$  external to the wire, as shown in Figure 1. The strength of this field varies as  $1/r$ , where  $r$  is the distance from the center of the wire. It also produces a magnetic field internal to the wire, which goes from  $1/r$  at the surface to zero at the center of the wire. For frequencies low enough that skin effect is negligible, the profile of magnetic field strength,  $H$ , versus radius,  $r$ , inside the wire is linear, as shown by the low-frequency line in Figure 2. Note that the field reaches a peak right at the surface of the wire, at radius  $r$ . Also note that a smaller diameter wire will create a larger peak  $H$  field at its surface, since the peak is proportional to  $1/r$ .

The magnetic and electric fields are vector fields [Indicated here in italic boldface type — Ed.] since at every point there is a magnitude and a direction to them. The magnetic field always forms closed paths or loops of magnetic flux, and the electric field always terminates perpendicular to a (perfect) conductor. The lines of flux shown in a field dia-

gram like Figure 1 follow the *direction* of the vector field, and the density of spacing between lines indicates the field magnitude. In general, the *magnitude* of the magnetic field along a given line is not constant; but in a symmetrical case like Figure 1, it is.

Flux is a smooth and continuous phenomenon, like swirling water. The lines in vector field diagrams merely indicate the trends of

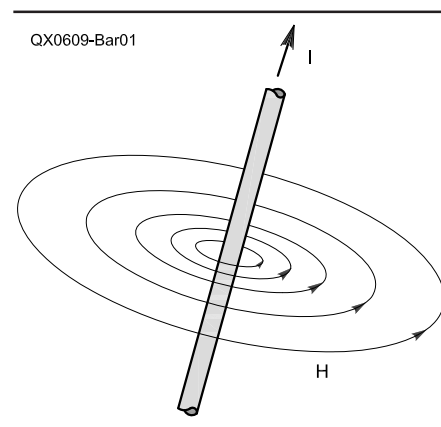


Figure 1 — Magnetic field around a current-carrying wire.

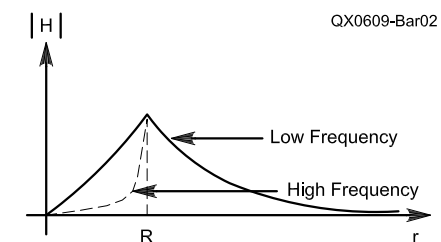


Figure 2 — Magnetic field strength variance with distance from center of wire.

the flux field. Flux does not actually occur as discrete lines.

The magnetic field  $H$  produces a magnetic flux density field,  $B$ , by the relationship  $B = \mu H$ , where  $\mu$  is the permeability of the medium in which the flux occurs ( $\mu$  is the product of the relative permeability  $\mu_r$  and the permeability of free space  $\mu_0$ ). Thus if a given  $H$  field passes through high-permeability material, a greater flux density will result than if it passes through air or copper, which have a  $\mu_r$  of 1. This flux density may vary along the path of an  $H$  field loop, too, depending on the permeability in portions of the path. Keep in mind that flux *density* is not the same as total flux — when flux density is integrated (summed) over an area, it results in the total flux passing through that area.

Refer to Figure 2 again. Since the wire and air have permeability of 1, this is also a picture of the flux density field,  $B$ , because of the current in the wire. The total flux produced is proportional to the integral of this  $B$  field, which is the area underneath the line in Figure 2 (between the line and the horizontal axis). The total flux inside the wire is constant for low frequencies, since the area of the triangle from 0 to any  $R$  is  $0.5 \cdot R \cdot 1/R = 0.5$ . The external total flux increases dramatically with shrinking  $R$ , however, since each reduction of  $\Delta R$  results in increased external flux of approximately  $\Delta R/R$  (the area under the added slice when  $R$  changes to  $R - \Delta R$ ). So, the total *external* flux produced by a wire at a given current increases greatly as the wire size is reduced.

Inductance is a measure of the magnetic flux *linking* a conductor relative to the amount of current producing the flux. Flux “links” a wire by circling it, like a link of a chain. Remember that flux always forms closed loops, so if a wire is linked by flux you can imagine pulling on the flux loop and having it tug the wire. This flux may arise from current in the wire itself, leading to self-inductance (also known as simply “inductance”), or from current in another wire, which results in mutual inductance. If more total flux links a wire for a given current, the inductance is greater. This applies to self-inductance or mutual inductance. Self-inductance is increased by using smaller wire, coiling the wire, or by increasing the permeability of the wire’s flux path. Mutual inductance is increased by causing more flux from one wire to link the other, that is, by bringing the wires closer or by increasing the permeability of the path of linking flux. Note that self-inductance is the sum of internal inductance from the flux inside the wire plus external inductance from flux outside the wire. These two components of self-inductance become important in understanding transmission line coupling below.

Bringing a current-carrying wire or coil closer to another increases the total flux link-

ing the two, so mutual inductance increases. In a single inductor, current through it causes voltage to be produced across it. When two inductors are linked by mutual inductance, current in one produces voltage in the *other* due to the flux from the one, which links the other.

Figure 3 shows two wires in cross section, with current entering the page in wire A and no current in wire B. The current in wire A causes the magnetic flux shown. None of the flux inside loop  $f_a$  links with wire B, so it contributes to the self-inductance of wire A only, and not to the mutual inductance between A and B. All the flux outside loop  $f_b$  does link with wire B, however, and contributes to both the self-inductance of A and the mutual inductance. Note that the mutual inductance gets much larger as the wires become very close, because of the  $1/r$  increase in flux magnitude. The amount of mutual inductance approaches the self-inductance as the wires approach perfect coupling.

Flux from more than one source present in the same region of space adds in a vector fashion at every point. This applies to magnetic or electric fields. The resulting field is still a vector field, and at every point the contributions from the individual fields are added vectorially. This may change the direction and magnitude of the resulting field considerably.

Imagine now that the same magnitude of current is present in both wires of Figure 3, with current coming out of the page in one and into the page in the other. This is simply a differential pair. The  $H$  field for one wire will be clockwise and counterclockwise for the other. Now when these wires are brought close together, the  $H$  fields add such that an intense *line* of  $H$  field is produced perpendicular to the line joining the centers of the wires, right between the wires. This is the region close to the wires where the individual  $H$  fields are the strongest and where the two fields point in the same direction. This  $H$  field line drops off rapidly away from the center line of the wires,

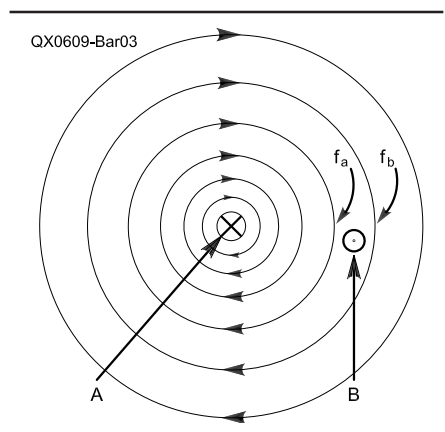


Figure 3 — Flux linkage from wire A to wire B.

since the tightly-curved  $H$  lines near the wires rapidly begin pointing in opposite directions. The combined  $H$  field is also nearly zero everywhere else, since the fields nearly cancel (they would perfectly cancel if the wires somehow occupied the same region in space).

The combined magnetic field of two close wires carrying differential current is markedly different from a single wire. An intense, short *line* of magnetic field is created *between* the wires, and the field is nearly zero everywhere else.

This altered field pattern, however, does not affect the individual inductance of the wires or their mutual inductance, because *inductance is a property that exists because of conductor geometry and the permeability of the surrounding medium only, regardless of any current or what the overall magnetic field pattern looks like*. That explains how ferrite-loading a coaxial cable can affect its inductance, despite the fact that its external magnetic field is nearly zero (see below).

A final note on inductance: For a particular wire carrying a particular current, a specific total  $H$  field is produced. If the permeability of the field path is increased, the inductance of the wire is increased. This is because the  $B$  field, and therefore the total flux produced by the current, increases. This is what happens when the wire is passed through a ferrite block, for example. This also *distorts* the  $H$  field, because the ferrite forms a low-reluctance path for the flux, so the field concentrates inside the ferrite since it is an easier path to follow.

## Transformer Operation

A transformer is an assembly of inductors (windings) that share flux — often nearly all of their flux. The windings are often wound such that their magnetic field paths are mostly through high-permeability material to ensure that as much flux as possible is shared, but transformers may also be built that use air or low-permeability cores. By adjusting the turns ratio between the windings, voltage and current can be stepped up or down.

The essentials of ideal transformer operation may be summed up as follows: *Changing winding current (ac) produces changing flux, which induces winding voltages related by the turns ratio*. Imagine a transformer core with two windings, and some changing shared flux existing in the core somehow, from some other source. (For now assume that all winding flux is shared and that there is no winding resistance.) These two windings will have a voltage induced across each of them by Faraday’s law.<sup>1</sup> These winding voltages will be *locked* together at their turns ratio by their common flux. If one winding has twice the turns of the other, it will have twice the voltage, too. If two

<sup>1</sup>Notes appear on page 36.

windings share the same changing flux, they *cannot* have a voltage that differs from their turns ratio, or Faraday's law would be violated.

This relationship also holds if one of the windings is producing the flux that links the two. The windings then have a primary/secondary relationship, with the primary creating the flux, but there is voltage induced in both primary and secondary in exactly the same way as before; it doesn't matter that the primary is also creating the flux. Faraday's law holds for both primary and secondary windings. See Appendix A for more on this.

Transformers sometimes appear as common-mode chokes, as in Figure 4. Current in the direction shown will produce flux in the core that adds (points in the same direction) and also links both windings. The result produces four times the inductance of a single side alone, since the mutual inductance adds with the winding inductance in this configuration. This inductance can be used as part of a filter to suppress common-mode currents.

The common-mode choke may also be called a "differential-mode pass-through." For pure differential-mode current,  $i_1 = -i_2$  as in Figure 4. That is, the same current enters the dotted side on one winding as leaves the dotted side on the other. In this case, the flux produced by one winding is exactly canceled by the other, resulting in zero core flux, and, by Faraday's law, zero voltage across each winding. A common-mode choke, therefore, simply passes differential-mode currents unimpeded.

In an actual circuit, both common-mode and differential-mode currents may be present. The common-mode choke is excellent for presenting high impedance to the common-mode portion, while not affecting the differential-mode currents.

In real transformers, the windings do not share 100% of their flux. Some amount of flux links one winding but not the other. This flux is accounted for in circuit modeling by "leakage inductance" in series with perfectly-coupled windings, as shown in Figure 5. The mutual inductance is usually many times greater than the leakage inductance, meaning that the great majority of flux produced is shared between the windings. Leakage inductance and winding resistance lead to winding voltages that are not exactly locked together at their turns ratio; but in well-coupled transformers, this is a good approximation.

In high-power transformer applications such as output baluns or matching networks for transmitters, the best possible coupling between windings and low winding resistance is critical to efficiency. The two loss mechanisms in high-power transformers are resistive power loss in the windings and core loss from eddy currents and hysteresis. Either of these will increase heating, sometimes to destructive levels, and reduce transformer

efficiency. Winding loss may be minimized by using large gauge wire or multiple smaller gauges, and core loss may be minimized by proper core material selection and keeping core flux as low as possible. If the windings are separated, the flux that couples them must pass through the core, leading to higher losses. If the windings are intimately wound as twisted pairs or close parallel lines, they will be coupled together with very little of the core required for the coupling flux path.

### Transmission-Line Fundamentals

It is important to realize that any pair of conductors carrying a signal and its return is a transmission line, at any frequency. There is no transition between "non-transmission-line" and "transmission-line" behavior. What happens at low frequencies is that the line becomes so electrically short (such a small part of a wavelength) that the line may be lumped into a single equivalent circuit representing its capacitance, inductance, and losses. Signal delay through the line need not be considered. And for many lines, such as the traces on a circuit board carrying moderate frequencies, those equivalent circuit elements have such a tiny effect that they may be neglected entirely. At higher frequencies, where the line length becomes an appreciable part of the signal wavelength (about  $1/10$  wavelength for most purposes), the effects of propagation time be-

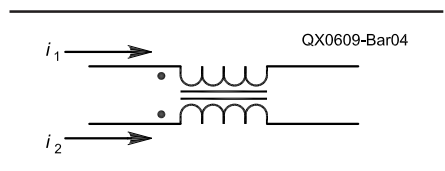


Figure 4 — Common-mode choke.

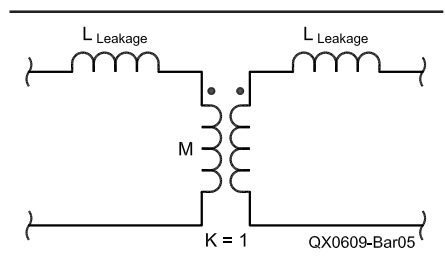


Figure 5 — Transformer model with leakage inductance.

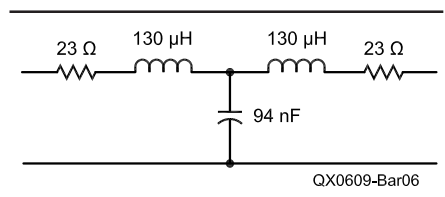


Figure 6 — Lumped-circuit equivalent of 1000 m of RG 58/U.

gin to be seen, which cannot be modeled by lumping the line into single elements. The line must then be represented by its distributed characteristics, and the interaction of forward and reverse waves must be considered.

As an example, consider 1000 m of RG-58 coaxial cable being used to transmit 60-Hz power. The wavelength of 60 Hz is about 3.5 million meters in RG-58, so even 1000 m of line may be lumped, since it is electrically short at the operating frequency. RG-58 cable specs are about 90 pF/m and 260 nH/m (more about what this means later), and it has a total low-frequency resistance of about 0.046  $\Omega$ /m. This is 90 nF, 260  $\mu$ H, and 46  $\Omega$  total for 1000 m, so when it is used to carry 60-Hz power, the cable may be represented as the symmetrical lumped circuit shown in Figure 6. The line resistance is far more than the inductive reactance at this low frequency, which is only about 0.098  $\Omega$ . Note that the characteristic impedance is still

$$Z_0 = \sqrt{260 \text{ mH} / 90 \text{ nF}} \approx 53 \Omega$$

(neglecting losses). If the load differs from 53  $\Omega$ , the line will be mismatched and both forward and reverse waves will exist, but they will reach equilibrium instantly with respect to 60 Hz. The waves combine exactly as they do at higher frequencies. *The forward and reverse wave analysis of long transmission lines reduces to circuit analysis for short lines, but the same physical phenomena are still taking place.*

Another important characteristic of transmission lines is that current does not "go out and come back." This water-in-a-hose analogy of electrical circuits is useful at dc, but it is not really the way ac electrical energy is transferred. Electrical energy is launched into a transmission line as a power wave of voltage and current, with current *pushed* onto the signal conductor at the same time it is *pulled* on the return conductor. The line is really only a guide for this electromagnetic power wave.

Now recall that any signal and its return forms a transmission line at any frequency and you will realize that even your reading light at 60 Hz receives its power in the form of a wave, guided by the power lines that feed it. An electromagnetic field is set up by your house wiring, and forms the actual medium through which the power is transferred. There is no low frequency limit where a source and return stop acting as a transmission line, but for short electrical lengths line characteristics may be lumped and propagation effects may be neglected. That means that for all HF work and much VHF, if the transmission line is less than a meter or so, there is negligible distributed transmission-line behavior. Transmission-line transformer windings a meter long become  $1/10$  wavelength at 21 MHz (assuming a velocity factor of 0.7), so this is where distributed transmission line effects will *start* becoming

evident. Shorter lines and lower frequencies will see negligible propagation effects and therefore their characteristics may be lumped.

### Progression from Loop to Shorted Transmission Line

Now it's time to consider that the ideal isolated current-carrying wire of Figure 1 does not really exist. Every current forms a circuit, so the best approximation we can make for Figure 1 is a large loop where the signal and its return are widely separated. The inductance of this configuration is mostly the self-inductance of the wire, but there is also a small mutual inductance between one side of the loop and the other. The magnetic field from one side of the wire drops off as  $1/r$ , but it is still present and links the other side eventually.

Now this loop can be pulled straight, forming two closely-spaced wires shorted at the end, with current in opposite directions on each side. This is obviously a shorted transmission line. The wire is still creating the same magnetic field as when it was a loop, the self-inductance of the wire is the same, but almost all the flux from each wire now links the other so the mutual inductance is greatly increased. The capacitance from one side of the line to the other has also increased, thus increasing both electrical and magnetic coupling.

The flux can be separated into two components: the flux that links both wires (contributing to mutual inductance), and the remaining *uncoupled* flux (internal to the wires plus a small shell around each wire). Figure 7 shows the inductance model for either the loop or the shorted line, where  $L_s$  means the residual self-inductance due to uncoupled flux and  $M$  means the mutual inductance due to coupled flux. In the loop configuration  $L_s$  is much larger than  $M$ , and in the closely spaced (transmission-line) configuration,  $M$  is much larger than  $L_s$ .

Now what total inductance,  $L_{tot}$ , do we see when we look into the open end of this shorted transmission line? It is clear in Figure 7 that the line is nothing more than a perfect common-mode choke (the mutual inductance) in series with the residual inductance  $L_s$ . Common-mode chokes are transparent to differential mode currents as we saw above, so the mutual inductance *does not appear at all* to the purely differential current from the open end.

*The only inductance seen at the open end of a shorted transmission line is that caused by the uncoupled portion of the flux from the individual conductors.*

This analysis is equivalent to the more common approach that calculates  $L_{tot} = L_{indiv} + L_{indiv} - 2M$ . Here  $L_{indiv}$  is the isolated self-inductance of an individual wire, which does not change as the wires are brought together. The mutual inductance  $M$  approaches  $L_{indiv}$ , as the wires get closer, causing  $L_{tot}$  to approach zero. The re-

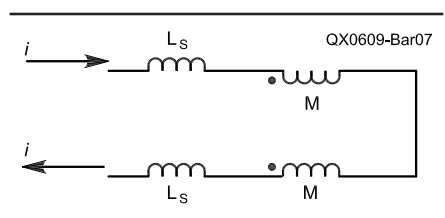
sult is the same, the only difference is the treatment of the flux that contributes to both  $L_{indiv}$  and  $M$ . I find it is helpful to consider the flux as composed of two parts: that which links the other wire and that which doesn't.

$L_{tot}$ , resulting only from the uncoupled line flux, is the inductance that is used to compute characteristic impedance, electrical length and so forth. It does not reveal the "hidden" mutual inductance, however, which is typically much larger.

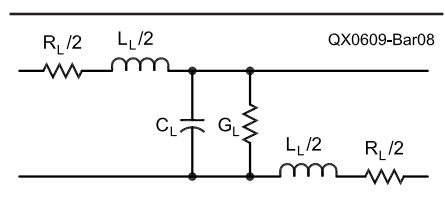
### A New Look at the Transmission Line

This leads to a revised model for the transmission line. The familiar model shown in Figure 8 (drawn symmetrically) is the equivalent circuit of a small portion of the line, where the subscript  $L$  means per-unit-length. In the limit, a transmission line consists of an infinite number of these infinitesimal sections. This model does not take into account the important quality of mutual inductance, though. The model should be drawn as in Figure 9, showing the distributed mutual inductance  $M$ . The inductance per length  $L_L$  is now the residual self-inductance due to uncoupled flux, and the mutual inductance in the model is perfectly coupled. Although the coupling is perfect, the mutual inductance is finite.

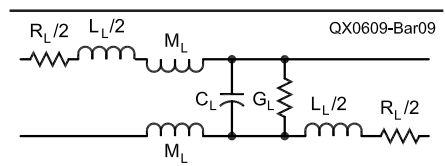
Now note that the inductive portion of this model is exactly the same as a 1:1 transformer turned sideways (the common-mode choke), shown in the standard configuration in Figure



**Figure 7 — Inductance of a loop or shorted line.**



**Figure 8 — Standard per-length transmission line model.**



**Figure 9 — Transmission line model with mutual inductance included.**

5. The line inductance  $L_L$  caused by uncoupled flux corresponds to the transformer leakage inductance, and the mutual inductance corresponds to the "ideal-transformer" part of the model, where the coupling is perfect.

*A transmission line may now be understood as actually being a transformer with the same flux coupling between windings as a conventional transformer, and with leakage inductance caused by the same phenomenon of imperfect winding coupling.*

The main difference is that there is more inter-winding capacitance and less leakage inductance in the transmission line version. In other words, the coupling is better. Also, transmission-line transformer turns ratios are usually limited to small integers, depending on how many individual conductors are incorporated into the line and how they are connected. Both types may be used longitudinally, where input and output are on the ends of the same winding and no dc isolation is provided; or conventionally, where the windings are in separate circuits and share most of their flux.

Now we can understand the invariance of characteristic impedance and electrical length when the line is surrounded by or wrapped around high-permeability material. The total inductance of the transmission line is only due to uncoupled flux. This flux is due to the internal  $H$  field of the wires themselves plus a small shell of  $H$  field external to each wire. Increasing the permeability of space outside this region does not increase the flux density due to these fields. Therefore the line inductance,  $L_{tot}$ , does not change and neither does the characteristic impedance or electrical length.

The magnetic field paths that surround both lines *do* pass through the high-permeability material, however. These external  $H$  lines are all lines that link both wires and therefore make up the line mutual inductance. *So placing high-permeability material around or near the entire line increases only the mutual inductance of the line.*

When a two-wire line is wrapped around a ferrite core,  $L_{tot}$  will only begin to depend on the core when the wires are separated from each other but make close contact with the core. In this case some of the uncoupled flux passes through the core and increases the wire's self-inductance, and therefore  $L_{tot}$ .

If the conductors of the line pass through high-permeability material *individually*, the  $B$  field of the uncoupled flux will be increased and the characteristic impedance and electrical length *will* be significantly affected. This will happen if the individual wires of a two-wire line pass through a binocular-type core, or if the dielectric of coax is ferrite-loaded, for example.

The ways this affects the model are twofold. First, there is simply more longitudinal inductance to the coupled portion of the line

because  $M = k\sqrt{L_1L_2}$ . The coupling factor  $k$  is nearly 1, and if mutual inductance  $M$  increases,  $L_1$  and  $L_2$  must both increase by the same factor as  $M$ . This is *perfectly coupled* inductance, remember, not simple series inductors. The second effect is that the coupling factor  $k$  actually increases if the mutual inductance is increased without affecting the inductance due to uncoupled flux. The coupling factor,  $k$ , is defined as the ratio of the perfectly coupled inductance of one winding to the uncoupled inductance of the winding. So,  $k$  goes from close to 1 because of the close physical coupling of the windings to *very* close to 1, perhaps 0.99, or nearly ideal.

If a transmission line is really a transformer, does the reverse also apply? That is, if a conventional transformer with separated windings is connected longitudinally (sideways, like a common-mode choke), will it behave like a transmission line? A sideways transformer has distributed inductance and capacitance throughout its structure, much like a transmission line. It seems that it would exhibit the same model behavior as Figure 9, so it might be expected to perform like a transmission line. What would be the differences?

First, the winding coupling would be reduced, since the windings would be physically separated and more uncoupled flux would be present. Filling the flux path between the windings with high-permeability material would help the coupling in the same way as with a transmission line, but it would not approach the same values of  $k$ . This leads to a higher uncoupled series inductance (leakage inductance), which reduces high-frequency performance.

The distributed inter-winding capacitance would be less, and less uniform. Unless the individual windings were done very carefully, this would also vary somewhat from unit to unit.

There is also a characteristic impedance associated with a sideways transformer, but it is higher because of higher  $L_{tot}$  and lower  $C$ , and less uniform with frequency because of the variable inductive and capacitive coupling.

There is more self capacitance across each winding relative to the inter-winding capacitance. When the windings are closely coupled as in a transmission line, the capacitance between the coupled wires will be much more than the capacitance between the ends of a given winding. The other winding(s) in a transmission line also act to partially shield a given winding from itself (or to completely shield in the case of a coax transformer). This additional winding self-capacitance leads to reduced high frequency performance and also spurious resonances.

Certainly when the winding lengths are electrically short, the sideways transformer and the transmission line will behave similarly,

given the same cores and winding inductances. The increased leakage inductance of the sideways transformer would cause greater losses and poorer high frequency response, and the less uniform inter-winding capacitance would cause unpredictable resonances also.

### Coax: A Special Line Topology

Some interesting line properties arise when one conductor is surrounded by the other, as in coaxial cable. The electric field between the inner and outer conductors is now completely confined to the space between them and perpendicular to both. More interesting is the magnetic field behavior. First note that for currents in the outer conductor (shield) only, there are *no magnetic flux lines* interior to it, as a consequence of Ampere's Law.<sup>2</sup> All flux lines due to current in the outer conductor are *external* to the outer conductor.

There is a magnetic field created by each conductor, external to the conductor, which varies as  $1/r$ , the distance from the coax axis. Since the conductors carry opposite currents and generate opposite magnetic fields, external to the outer conductor the magnetic fields are equal and opposite and ideally cancel to zero.

That requires the current to be exactly equal and opposite, the conductors to be exactly coaxial, and for current distribution to be exactly symmetrical, which of course is never actually achieved. Even in practical coax, however, the electric field is well-contained and the external magnetic field is nearly zero. Because of the absence of flux loops from the outer conductor in its interior, *the self-inductance of the outer conductor exactly equals the mutual inductance between the two conductors.*

*Every* flux line the outer conductor produces is exterior to it and links both the inner conductor and itself. Thus, every flux line from the outer conductor contributes to both its self-inductance and the mutual inductance. This is not true of the inner conductor; some of the flux it produces does *not* link the outer conductor and therefore contributes only to the inner conductor self-inductance. This unlinked flux exists between the inner and outer conductors, and is due to the inner conductor only.

What this means is that *no longitudinal voltage will exist* on the outer conductor due to inductance, if the inner and outer conductor currents are equal and opposite! Any tendency to create longitudinal voltage on the outer conductor due to current through its inductance will be exactly cancelled by the same current on the inner conductor due to the same mutual inductance. As electrical power propagates along the coax, there will be a voltage on the inner conductor relative to the outer and relative to the source (if it is electrically long), but there will be no voltage difference between the source and load ends of the outer conductor due to inductance.

For long lines or high current, a longitudinal voltage can develop on the outer conductor due to finite resistance. This is *not* an inductive effect, however.

With this in mind, coaxial cable should *not* be used in transformers which require the outer conductor to develop a longitudinal voltage, when the inner and outer conductors carry equal and opposite currents. For any use of coax in a transformer this effect must be considered.

### Skin and Proximity Effects

Most RF buffs know that wire resistance becomes greater with higher frequency due to skin effect. Skin effect causes high-frequency current in a conductor to move toward the outer regions (skin) of the conductor, so there is greater current density and a larger effective resistance. What is not commonly known is that the skin effect becomes significant at surprisingly low frequencies, and that skin effect also causes total inductance to *decrease*.

Skin effect is caused by variation in inductance across the cross section of the conductor due to the magnetic field *inside* the conductor. The current toward the center of the conductor is linked by more flux than the current toward the outside, so the portion of the conductor toward the center has higher inductance than that toward the outside. The current distributes itself to minimize the total *impedance* it must pass through, so at higher frequencies this means following a path with reduced inductance. The internal inductance difference between the center and the skin is negligible at lower frequencies but becomes very significant as the frequency increases. In the microwave region almost all the current is crowded into a skin only microns thick.

While skin effect causes current to follow the path of least inductance in one conductor, proximity effect causes current to follow the path of *greatest mutual inductance* between two conductors of a transmission line. The general rule for both effects is that *current will tend to distribute itself so that total inductance is minimized.*

As we saw above, when transmission line wires come closer together their total inductance drops due to increasing mutual inductance. This process simply continues with proximity effect, drawing current within the conductors themselves closer. Figure 10 shows how current distributes itself in two close wires of a transmission line, concentrating toward the middle so more flux from one wire couples with the other. Skin effect also accompanies proximity effect, pushing current toward the wire periphery.

The current does not abruptly cut off due to skin effect as the center of the conductor is approached, but it falls off exponentially. The *skin depth* is defined as the point at which

the current has fallen to 1/e or 37% of its maximum at the surface, so most of the current is confined to one skin depth and nearly all of it is within three skin depths.

The radius of three skin depths for copper is shown in Table 1 for various frequencies, with the corresponding gauge of wire this represents.

Table 1 shows that even at 1 MHz, nearly all the current is in a shell the thickness of the radius of AWG 26 wire. The copper near the center of any wire larger than that will not carry any significant current.

That does not mean that high-power transformers at 1 MHz may use AWG 26 wire, however! High-frequency resistance will continue to drop with larger wire since the total cross-sectional area of the skin region will increase. This lower resistance is necessary to keep losses low in high-power applications. The center portion of the wire could be hollowed out, however, and there would be little effect.

Skin effect also causes internal inductance to be reduced at high frequency. Since the current is pushed to the periphery of the conductor, the generation of magnetic flux is too. This causes the  $H$  field profile inside the conductor to assume the shape shown in Figure 2 for high frequencies. Since the total flux inside the conductor is the area under this curve between 0

and  $R$ , it is clear that less total internal flux is being generated for the same amount of current. Thus the internal inductance drops.

The external field of a conductor is not affected by skin effect, however. With a given transmission line this decrease of internal inductance with the same mutual inductance will cause the coupling factor to increase with frequency due to skin effect.

Skin and proximity effects on inductance are difficult to calculate, but the trends are helpful to understand. They can cause differences in measured values of inductance and coupling of 15 to 20% or so, even in the HF region.

### Line Measurements

I measured a 1-m length of RG58-A/U and a 1-m length of twisted pair stripped from some CAT-5 cable (solid AWG 24 insulated wires), treating these transmission lines as if they were transformers. In other words, for the coax the center wire was the primary and the shield the secondary; and for the twisted pair, one wire was primary and the other secondary. I arranged each of these lines in one large loop for the measurements. I measured the coupling factor  $k$ , total inductance  $L_{tot}$ , the mutual inductance  $M$ , and the characteristic impedance  $Z_0$  under two conditions: 1) without any ferrite loading, and 2) with seven blocks of ferrite snapped around the line. These ferrite blocks totaled 22 cm in length, so they surrounded only a small portion of the line length.

The results are shown in Table 2. Mutual inductance between the two conductors of the transmission line is clearly a significant factor, and the coupling factor  $k$  and the mutual inductance  $M$  both increase significantly with ferrite loading. However,  $L_{tot}$  and  $Z_0$  remain the same because the uncoupled inductance

doesn't change. The procedures for this measurement are given in Appendix B.

### Analyzing the Transmission-Line Transformer

Now it is clear that the transmission line and the transformer are very closely related. *An electrically-short transmission line is really just another method of constructing a transformer, and winding this transmission line around a ferrite core simply increases the mutual inductance.* It is instructive to examine some transformer configurations using the tools developed here.

There has been a distinction made between the low-frequency behavior of these transformers, where the "...choking reactance becomes insufficient...", and higher frequencies. A separate analysis is made for low frequencies. The analysis of these transformers does not change at the low frequencies, however. From dc until the winding length is no longer electrically short, the analysis is the same: they are treated as lumped-circuit transformers. The distinction instead should be made between lumped and distributed analysis. This transition occurs at 20 to 100 MHz or so, depending on winding electrical length.

Another common error is the failure to recognize the close magnetic coupling between the windings of the transmission line transformer, and the fact that this leads to the winding voltages being equal. Analysis should begin with this fact and derive other voltages and currents from there.

It would be helpful to go through several typical examples to see how these transformers should be approached. This begins with the basic building block circuit as shown in Figure 11. Dots have been added to the figure

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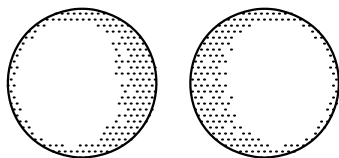


Figure 10 — Current distribution due to proximity and skin effects in a two-wire line.

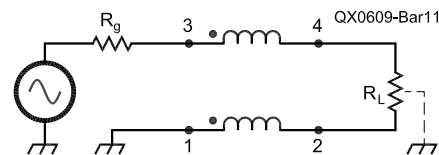


Figure 11 — Basic transmission line transformer building block (and "balun").

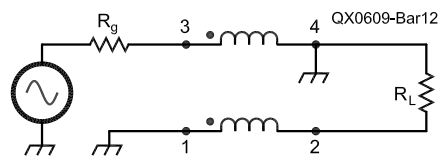


Figure 12 — Phase inverter.

Table 1  
Skin Depth Versus Frequency

	100 kHz	1 MHz	10 MHz	100 MHz	1 GHz
Three skin depths (mils)	25	7.8	2.5	0.78	0.25
AWG equivalent	16	26	36	46	—

Table 2  
Coupling Factor and Mutual Inductance for Transmission Lines

	$k$	$M$ ( $\mu H$ )	$L_{tot}$ ( $\mu H$ )	$Z_0$
Plain coax	0.84	0.86	0.28	52.4 $\Omega$ $\angle -1.7^\circ$
Coax plus ferrite	0.98	11.90	0.28	52.5 $\Omega$ $\angle -1.7^\circ$
Plain twisted pair	0.78	0.99	0.58	122 $\Omega$ $\angle -0.1^\circ$
Twisted pair plus ferrite	0.98	12.20	0.58	121 $\Omega$ $\angle +0.1^\circ$

to emphasize the polarity and the transformer-like nature of the two-wire line. I will use the notation  $V_{xy}$  to mean the voltage at node  $x$  with respect to  $y$ , and  $v_x$  to mean the voltage at node  $x$  with respect to ground. There are four cases to consider: grounding node 4 (phase inverter), as the circuit is shown (“balun”), grounding pin 2 (“delay line”), and connecting nodes 3 and 2 (“bootstrap”).

#### Phase Inverter, Node 4 Grounded

See Figure 12. First, disconnect the load and examine the magnetizing current (see Appendix A). The source provides magnetizing current directly through  $R_g$  and the top winding. There will be a voltage  $V_{34}$  across the top winding from voltage divider action between the reactance of the winding and  $R_g$ . Because of the flux which couples the two windings,  $V_{34}$  always equals  $V_{12}$ . Since nodes 1 and 4 are grounded, this places  $v_2$  at  $-v_3$  potential and provides a phase inverter.

Now when  $R_L$  is connected between nodes 4 and 2, current will be pulled through it because the load voltage is  $-v_3$ . The lower winding will provide this current since node 4 is grounded, but because of the restoration of magnetizing flux in the transformer (see Appendix A) this same current will be pulled from the source strictly through flux coupling, and has nothing to do with distributed transmission line behavior.

For frequencies where the line is electrically short, reducing  $R_L$  and reducing the winding reactance have the same effect. In both cases there is more current through  $R_g$ , either because of transformer-coupled load current or because of reduced reactance (increased magnetizing current) through the top winding. In either case, the drop through  $R_g$  causes  $v_3$  to drop and the load voltage to do the same, because the longitudinal winding voltages are locked together by their shared flux. This circuit performs an exact phase inversion down to audio frequencies even with typical RF windings and cores, but the output amplitude falls off because of the decrease in reactance of the top winding.

The low-frequency cutoff of the transformer is the point where the winding reactance equals the parallel combination of  $R_g$  and  $R_L$ . Remember that the winding reactance is dominated by the mutual inductance which increases with ferrite loading, so the  $-3$  dB frequency can be lowered considerably by adding ferrite.

It is not necessary to require a matched load for this circuit when the windings are electrically short, either for analysis or for operation. Simple circuit analysis is all that is required, remembering that the

winding voltages remain the same.

#### “Balun”

The circuit of Figure 11 is also known as a Guanella current-mode balun. It is not really a balun, however. Neither currents nor voltages are forced to balance by this circuit. It is, in fact, a common-mode choke. There is no magnetizing current with the load disconnected, so the transmission line currents are fully differential. This means there is no core flux, therefore no longitudinal voltage on the windings, and  $v_2$  is at ground potential. This does not change when ferrite is added to the line, increasing the common-mode choke reactance.

The common-mode choke must have some common-mode impedance following it to affect common-mode signals. The circuit as shown with no ground at the load does not provide that, as it is a purely differential load. Therefore, the load as a whole will become more isolated from the source as the reactance of the choke increases, but in the absence of external ground paths,  $v_2$  will remain at ground potential. If parasitic or direct grounding is placed in the load somewhere, this provides a common-mode impedance and works with the common-mode choke and the source to determine the load voltage. With the choke reactance high enough, the load may be grounded at its midpoint and it will become truly balanced. The choke has not created this balance, however. It has isolated the load enough that it may be made to balance by the midpoint ground connection.

Note that whatever voltage drop  $V_{34}$  occurs across the top winding caused by choke reactance, load, or common-mode current will also be induced across the lower winding ( $V_{12}$ ) by transformer action. This means the total load voltage  $V_{42}$  will remain the same regardless of where the ground point might appear in the load.

#### “Delay Line”, Node 2 Grounded

This is a good example of the importance of mutual inductance in the transmission line model. Without mutual inductance (such as the model of Figure 8), the circuit of Figure 13 does indeed form a delay line. When mutual inductance is considered, however, this is no longer true. Grounding node 2 is then the same as shorting the secondary of a well-coupled transformer. The short is passed directly onto the primary, in this case the top winding.

Here is another way to look at it. Imagine that the transmission line has a large delay,  $\frac{1}{2}$  cycle, so when  $v_3$  is at a positive peak,  $v_4$  is at a negative peak. This maximizes the voltage across the upper winding. Now if the lower winding is shorted by grounding node

2, the upper winding is forced to the same voltage, that is, zero. No voltage appears across the upper winding and no delay exists. Once again, remember that the two windings share nearly all their flux and must maintain the same voltage.

This analysis only applies to wire lines. A coaxial line will act as a delay line in this configuration if it is the shield that is grounded at both ends, since the shield does not display any longitudinal voltage. See the discussion on coaxial lines above.

#### Ruthroff Unbalanced Impedance Transformer

Look at Figure 14A, sometimes called a “bootstrap.” When nodes 2 and 3 are connected together and  $R_L$  is connected from node 4 to ground the familiar Ruthroff unbalanced 1:4 impedance converter is obtained. This circuit has been described as “summing a direct voltage with a delayed voltage that traversed a single transmission line.” There is no such thing as a “single transmission line,” meaning one without a return, however, and delay has nothing to do with this circuit except to reduce its performance. It is misleading to analyze it in this way. By looking at the voltages across the nodes, we can see that  $V_{43} = V_{21}$  because of flux coupling, so  $v_4$  always equals  $2v_2$ . The input voltage at nodes 2 and 3 is therefore doubled at node 4, so the configuration is a times two voltage step-up transformer. The power consumed by the load is drawn from the source,

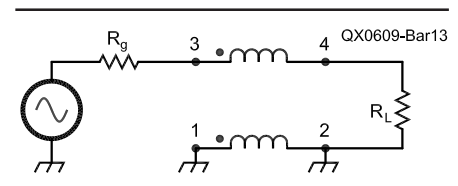


Figure 13 — “Delay line.”

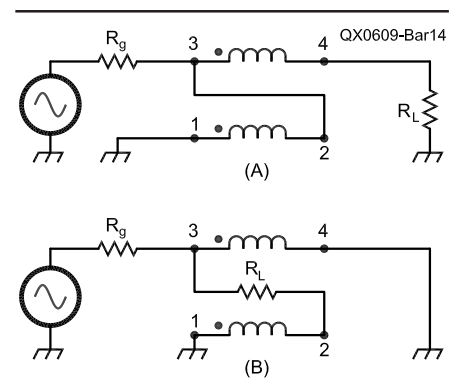
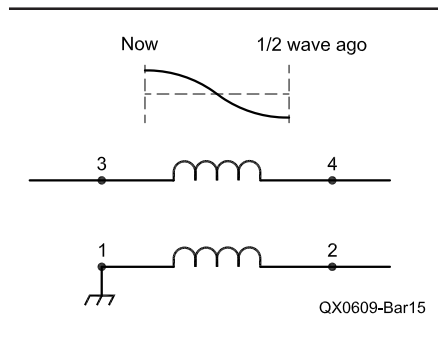


Figure 14 — Ruthroff unbalanced and balanced impedance converters.



**Figure 15 — Delay through a half-wavelength-long transformer.**

which is  $V_{load}^2 / R_L = (2v_2)^2 / R_L = v_2^2 / (1/4R_L)$  — in other words, as if a load of  $1/4R_L$  were connected directly to  $v_2$  without the transformer. Thus the impedance  $R_L$  is transformed to  $1/4$  its value, as seen at  $v_2$ .

This 1:4 impedance transformation does not depend on load matching as long as the electrical length of the windings is short. Any load will be transformed to  $1/4$  its value as seen at  $v_2$ . This effective load will draw current through the source and cause a voltage drop through  $R_g$ , depending on the value of the load. When the reactance of the windings is low, there will also be magnetizing current through the lower winding and will cause an additional input-voltage drop.

This transformer (and the next example) actually begins to *degrade* when the windings start acting like transmission lines (that is, when delay starts to become a factor). The voltage wave created by the source at  $v_3$  is connected to what the wave *was* some fraction of a wavelength ago at  $v_2$ . Figure 15 shows the distributed voltage wave across a transmission line which is  $1/2$  wavelength long. The source voltage at the instant shown is at the positive peak, while the voltage at the far end of the line (nodes 4 and 2) is at the negative peak. If nodes 2 and 3 are connected now the transformer ceases to function, but it will be impaired well before the windings become  $1/2$  wavelength long. By this reasoning these configurations should not be considered transmission line transformers at all!

### Ruthroff Balun

Figure 14B shows an actual balun, where the voltage across the load is forced to balance with respect to ground. This circuit is very similar to the phase shifter discussed above, where  $v_3$  is seen to equal  $-v_2$ . So twice the source voltage at  $v_3$  appears across the load, and by the same analysis as the Ruthroff unbalanced configuration above, the load appears as  $R_L / 4$  to node 3. So, this is indeed a

1:4 impedance-converting balun. Beware of the performance degradation discussed above as it starts to assume distributed line characteristics.

### A Recap

I have answered my original questions and then some:

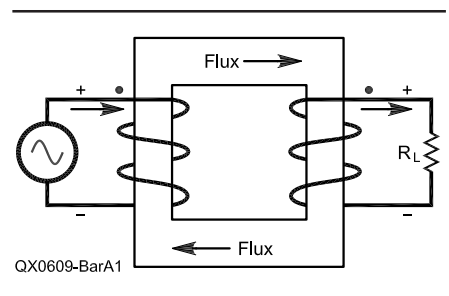
- Conventional transmission line models are missing a crucial element for transformer analysis: the distributed *mutual inductance*. This is much larger than the uncoupled inductance of the model and has a significant effect on the line's behavior when used as a transformer.
- Transmission line transformers, when the windings are electrically short, are not qualitatively any different from conventional transformers. They are simply very well coupled, which reduces leakage inductance and provides uniform distributed capacitance. There is no distinction between "transmission-line mode" and "conventional-transformer mode" in transformer operation.
- Characteristic impedance and electrical length don't change with ferrite loading because they depend only on the *uncoupled* flux of the individual line conductors. This is almost entirely internal flux when the line is tightly coupled.
- Ferrite loading a line increases the *mutual inductance only*, which also improves the coupling factor. This improves both high and low frequency performance, if the ferrite retains its permeability sufficiently at the high frequencies.
- Using the coupled-windings analysis presented here, a clear distinction is seen between the common-mode choke and the balun. These have been confused in the literature.
- The analysis differs between *lumped* and *distributed* transformer behavior, not between low and high "choking reactance."
- Inductance is a property of conductor geometry and the permeability of the space surrounding the conductor only. It doesn't depend on the fields present.

### Acknowledgments

I wish to gratefully acknowledge three of the brightest engineers I know for their helpful review of this article: Larry Brasfield, George Keilman, and Fred Telewski, WA7TZY.

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**Figure A1 — Voltage, current, and flux relationships in a transformer.**

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### Appendix A: More Transformer Theory

In conventional transformer operation, ac current from a source passes through a primary winding and generates flux. This flux links with one or more secondary windings, and, in accordance with Faraday's law, induces a *voltage* in the secondary. What is often overlooked is that this flux also generates voltage in the *primary* in exactly the same way. The same flux links both windings, and Faraday's law applies to all windings regardless of where the flux originates.

The polarity of the voltages induced is critical. Lenz's law states that the voltage induced in a conductor by changing flux will be such that it would produce current that would *oppose* the flux that generated it. This sounds rather confusing, but consider Figure A1, showing one instant of time in transformer operation. The voltage source shown produces current into the dotted end of the primary, which produces flux in the direction shown. Check this with the right-hand rule: with fingers curling in the direction of the winding current, the thumb points in the direction of the flux generated. This flux passes through the secondary in the direction shown and induces a secondary voltage as shown. Use your right hand to verify that current from this induced voltage into  $R_L$  creates flux in the *opposite* direction from that which produced the voltage.

Now this same flux also creates voltage in the primary which would also tend to create opposing flux. This induced primary winding polarity is the same as the source polarity. Imagine the source replaced by a resistor, and the same flux somehow present

in the core (from some other source). Current would flow *out* of the dotted primary end and thus create flux in the opposite direction of that shown. Thus the primary presents an induced voltage to the source with the same polarity as the source.

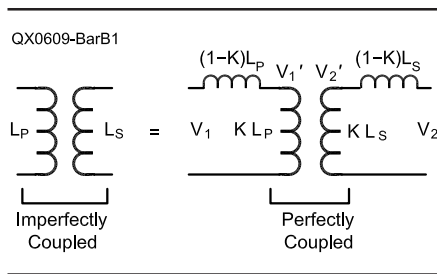
So how can there be any current in the primary if a voltage is present on the primary which opposes the source? Wouldn't this result in zero current? Well, imagine that this were so. With no primary current, there would be no flux, and no induced voltage in the primary, which would then produce current! So there is obviously some induced, opposing primary voltage *less* than the source voltage, which allows just the right amount of current to produce the necessary induced voltage. This is really just the regulating mechanism that defines inductive reactance. That is, in any inductor a voltage is induced such that the current that flows is equal to the applied voltage divided by the reactance. For higher frequency or larger inductance, the induced voltage is closer to the applied voltage and therefore there is less current.

This is a complicated introduction to some simple rules that result. There are really only two rules to remember with ideally coupled transformer operation:

- The voltage ratio of each winding (in the absence of winding resistance) will be equal to the turns ratio due to the flux sharing as described above. The polarity is such that the dotted ends (ends entering the core from the same side) will have the same polarity.
- The winding currents need only satisfy two requirements: core magnetization and loss, and load current. These can be determined by disconnecting the load to determine the magnetizing current, and connecting it to determine the effect of the load.

Core magnetization is another term for the current drawn by the finite inductive reactance of the primary winding. Core loss is power drawn from the source to overcome hysteresis and eddy current losses, and applies to ferrite or iron cores. The above rules may be used to determine the behavior of the ideally coupled portion of the transmission line model (identical to a transformer), and the remainder of the model may be added to refine the analysis.

It is also interesting to note that the flux present in the core of a transformer is only that due to core magnetization (neglecting losses), *regardless of load current*. Changing the load current does not change the core flux. To visualize this, refer again to Figure A1, where a voltage source is shown connected directly across the primary. First disconnect the load  $R_L$ . Now there is no current in the secondary, and the only primary current is that drawn by the reactance of the primary winding, which depends on the fre-



**Figure B1 —Equivalent circuits for transformer coupling measurement.**

quency of operation and the winding inductance. This is the magnetizing current. If the winding reactance is high the magnetizing current will be low, and vice-versa.

Now when a load is connected to the secondary, current is drawn which tries to create flux in the *opposite* direction of the flux established by the magnetizing current. The primary won't allow this to happen, though, since the flux and the primary voltage are locked together by Faraday's law. If the flux changes, the primary voltage must change, but this cannot happen with a voltage source connected to it. Instead, the primary supplies more current, which generates more flux to counteract the flux opposition and thereby restore the flux to the original magnetizing level. This happens effectively instantly (that is, there is no phase shift between primary and secondary currents). So, the load current is immediately drawn from the primary (in proportion to the turns ratio) without changing the core flux. You can think of the flux as a rigid gearbox connecting the windings that transmits the load to the energy source with a fixed gear (turns) ratio.

## Appendix B: Transmission Line Measurements

The transmission line measurements above were made on an HP 8151A vector network analyzer with an impedance test fixture. Open/short/load compensation was performed for maximum accuracy. The measurements were made at 2 MHz to avoid excessive skin effect, and a sweep from 1 to 4 MHz was performed to verify that reasonable impedance behavior was seen.

Characteristic impedance  $Z_0$  was determined by measuring  $Z_{short}$  and  $Z_{open}$  with the far end of the line shorted and opened respectively.  $Z_0$  is then found from:

$$Z_0 = \sqrt{Z_{open} \cdot Z_{short}} \quad (\text{Eq B1})$$

$L_{tot}$  is determined from the imaginary part of  $Z_{short}$ .

Transformer coupling was determined by the following means, illustrated in Figure B1:

$L_p$  = measured midrange inductance of primary, secondary open

$L_s$  = measured midrange inductance of secondary, primary open

$\frac{v_2}{v_1}$  = measured midrange voltage ratio, secondary open,  $v_1$  actual loaded voltage across the primary.

For the voltage ratio test,  $v_1'$  is divided from  $v_1$  by the series primary inductors.

$$v_2' = \sqrt{\frac{kL_s}{kL_p}} \cdot v_1' = \sqrt{\frac{L_s}{L_p}} \cdot v_1' \quad (\text{Eq B2})$$

The coils are perfectly coupled, and there is no voltage drop across the secondary leakage inductor.

$$v_1' = \frac{kL_p}{kL_p + (1-k)L_p} \cdot v_1 = k \cdot v_1 \quad (\text{Eq B3})$$

$$v_2' = v_2 = \sqrt{\frac{L_s}{L_p}} \cdot v_1' = kv_1 \sqrt{\frac{L_s}{L_p}} \quad (\text{Eq B4})$$

so

$$k = \frac{v_2}{v_1} \cdot \sqrt{\frac{L_p}{L_s}} \quad (\text{Eq B5})$$

and by definition

$$M = k \sqrt{L_p L_s} = \frac{v_2}{v_1} \cdot L_p \quad (\text{Eq B6})$$

## Notes

<sup>1</sup>Faraday's law is one of Maxwell's equations, and states that voltage is induced in a conductor in proportion to the rate of change of the total flux linking it. That is, higher amplitude or higher frequency current will result in a higher induced voltage, as will more total flux linkage — by coiling the wire around the flux path, for example.

<sup>2</sup>Ampere's Law is another of Maxwell's equations, and states that the net magnetomotive force in any closed loop (that is, the summation of the tangential  $H$  field around the loop times the loop length) is equal to the net current passing through the loop. Any flux line has a non-zero magnetomotive force since it points in the direction of  $H$  for the entire loop (that is,  $H$  is always tangential to it and in the same direction). So any closed loop which does not enclose net current cannot be a flux line, thus no flux lines exist in the interior of the coax outer conductor due to current in the outer conductor. An  $H$  field *may* exist inside a loop which encloses zero net current, but this field cannot produce a magnetomotive force around the loop.

## Biographical Notes

Gerrit Barrere, KJ7KV, is a consulting electronic engineer ([www.exality.com](http://www.exality.com)) living in Shoreline, near Seattle, WA. He was first licensed in 1995. His professional and Amateur Radio interests include transformers, RF amplifiers, receivers, and signal processing, and he has done much work in medical ultrasound and sonar applications. Gerrit enjoys volunteering, music, reading, and cooking.

