Measuring Transformer Coupling Factor $k$

G. Barrere – Exality Corporation

A transformer with individual winding inductances $L_1$ and $L_2$ has mutual inductance $M$ between the windings. Transformer terminal equations are:

$$ V_1 = j \omega (L_1 \cdot I_1 + M \cdot I_2) \quad (1) $$
$$ V_2 = j \omega (M \cdot I_1 + L_2 \cdot I_2) \quad (2) $$

If winding 2 is shorted, $V_2$ becomes zero so equation (2) becomes:

$$ 0 = j \omega (M \cdot I_1 + L_2 \cdot I_2) $$

Solving for $I_2$:

$$ I_2 = -\frac{M \cdot I_1}{L_2} \quad (3) $$

Substitute equation (3) into equation (1):

$$ V_1 = j \omega (L_1 \cdot I_1 - \frac{M^2 \cdot I_1}{L_2}) = I_1 \cdot j \omega (L_1 - \frac{M^2}{L_2}) \quad (4) $$

If you define

$$ L_S = L_1 - \frac{M^2}{L_2} \quad (5) $$

then equation (4) is in the form

$$ V_1 = I_1 \cdot j \omega L_S $$

which is simply the voltage and current relationship of an inductor. $L_S$ is therefore the inductance measured across $L_1$ with winding 2 shorted. Solving equation (5) for $M^2$:

$$ M^2 = L_2 (L_1 - L_s) \quad (6) $$

The definition of transformer coupling factor $k$ is

$$ k = \frac{M}{\sqrt{(L_1 \cdot L_2)}} $$

or
\[ k^2 = \frac{M^2}{L_1 \cdot L_2} \quad (7) \]

Substitute equation (6) into equation (7):

\[ k^2 = \frac{L_2(L_1 - L_s)}{L_1 \cdot L_2} = \frac{(L_1 - L_s)}{L_1} = 1 - \frac{L_s}{L_1} \]

or

\[ k = \sqrt{1 - \frac{L_s}{L_1}} \quad (8) \]

\( L_1 \) is the inductance measured across \( L_1 \) with winding 2 open and \( L_s \) is the same measurement with winding 2 shorted. \( K \) is determined by inserting these inductance values into equation (8).