

Generating Arbitrary Phase and Amplitude from I and Q Sum

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A sinusoid of arbitrary phase and amplitude relative to a reference sinusoid may be generated if the in-phase (I) and quadrature (Q) components of the reference are available. In other words, the sinusoid

$$Z \cos(\alpha + \beta)$$

may be created from $\cos \alpha$ (the I component) and $\sin \alpha$ (the Q component) by forming the scaled sum $X \cos \alpha + Y \sin \alpha$ with the proper choice of X and Y. This represents a sinusoid which leads the reference sinusoid $\cos \alpha$ by β degrees.

Start with the trig identity:

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

Multiply by Z:

$$Z \cos(\alpha + \beta) = Z \cos \alpha \cos \beta - Z \sin \alpha \sin \beta$$

Group terms on the right side:

$$Z \cos(\alpha + \beta) = (Z \cos \beta) \cos \alpha + (-Z \sin \beta) \sin \alpha$$

Now it is clear that

$$\begin{aligned} X &= Z \cos \beta \\ Y &= -Z \sin \beta \end{aligned}$$

For example, to create a sinusoid which is .25 times the amplitude of the reference and leads it by 127 degrees, use

$$\begin{aligned} X &= .25 \cos(127) = -.150 \\ Y &= -.25 \sin(127) = -.200 \end{aligned}$$

which is to say

$$.25 \cdot \cos(\alpha + 127) = -.150 \cdot \cos \alpha - .200 \cdot \sin \alpha$$