

Coordinates of the Reciprocal of a Vector

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Given a vector V directed from the origin to the point (x, y) , what are the coordinates of $\frac{1}{V}$? By

definition, the inverse of a vector with magnitude M and angle θ , written as $M \angle \theta$, is

$\frac{1}{M} \angle -\theta$. We have $V \equiv (x, y) = M \angle \tan^{-1} \frac{y}{x}$ where $M = \sqrt{x^2 + y^2}$, so

$$\begin{aligned} \frac{1}{V} &= \frac{1}{M} \angle -\tan^{-1} \frac{y}{x} \\ &= \left(\frac{1}{M} \cos \left(\tan^{-1} \frac{y}{x} \right), \frac{1}{M} \sin \left(\tan^{-1} \frac{y}{x} \right) \right) \text{ ordered pair of } x, y \text{ coordinates} \\ &= \left(\frac{1}{M} \cdot \frac{1}{\sqrt{1 + \left(\frac{y}{x} \right)^2}}, \frac{1}{M} \cdot \frac{\left(\frac{y}{x} \right)^2}{\sqrt{1 + \left(\frac{y}{x} \right)^2}} \right) \text{ from trig identities} \\ &= \left(\frac{1}{M} \cdot \frac{x}{\sqrt{x^2 + y^2}}, \frac{1}{M} \cdot \frac{-y}{\sqrt{x^2 + y^2}} \right) \\ &= \left(\frac{x}{M^2}, \frac{-y}{M^2} \right) = \left(\frac{x}{x^2 + y^2}, \frac{-y}{x^2 + y^2} \right) \end{aligned}$$

Either of the final results is quite simple, especially if M is known beforehand.